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Effects of ^{8}B size on the low-energy ^{7}Be$(p,\gamma)^8$B cross section

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Abstract

We calculate several "size-like" ^{8}B observables within the microscopic three-cluster model and study their potential constraints on the zero-energy astrophysical $S_{17}(0)$ factor of the ^{7}Be$(p,\gamma)^8$B reaction. We find within our three-cluster model that a simultaneous reproduction of the experimental data for the ^{8}B radius and quadrupole moment and of the ^{8}B-^{8}Li Coulomb displacement energy implies $S_{17}(0) = (23 - 25)$ eV b.

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I. MOTIVATION

The ^{7}Be$(p,\gamma)^8$B reaction is currently considered to be one of the astrophysically most important nuclear reactions, as its low-energy cross section determines the high-energy solar neutrino flux [1]. Recently there has been a great deal of experimental and theoretical activities investigating this process. The low-energy cross section has been studied directly by using a radioactive ^{7}Be target and a proton beam [2], in inverse kinematics by using a ^{7}Be beam and a proton target [3], indirectly from the Coulomb dissociation of ^{8}B [4], and by extracting the $^7\mathrm{Be}+p$ nuclear vertex constant from the ^{7}Be$(d,n)^8$B reaction [5]. On the theoretical side some effects of ^{7}Be deformations [6] and three-body dynamics [7] have been studied, and efforts to understand the nuclear vertex constant have been made [8]. In Ref. [9] we have shown that the zero-energy cross section of the ^{7}Be$(p,\gamma)^8$B reaction scales linearly with the, unfortunately yet unknown, quadrupole moment of ^{7}Be. In the present paper we extend this study and investigate the relation between the zero-energy cross section and several ^{8}B "size" properties.

In our approach we study the ground state properties of ^{7}Be and ^{8}B as well as the ^{7}Be$(p,\gamma)^8$B reaction cross section consistently within the microscopic eight-body $^4\mathrm{He}+{}^3\mathrm{He}+p$ cluster model. As this model has been discussed before we refer the reader to Refs. [10,9] for details of the theoretical background. As customary in nuclear astrophysics we define the cross sections in terms of the astrophysical S factor

$$S(E)=\sigma(E)E\exp\Big[2\pi\eta(E)\Big],\qquad \eta(E)=\frac{Z_1Z_2e^2}{\hbar v},\tag{1}$$

where Z_1, Z_2 are the charges of the two colliding nuclei, and v is their relative velocity.

At low, and in particular, at solar energies the ^{7}Be$(p,\gamma)^8$B reaction is highly peripheral, which means that only the external parts of the bound- and scattering wave functions contribute to the radiative capture cross section [11]. The external wave functions are known with the exception of the asymptotic normalization constant, $\bar{c}$, of the ^{8}B bound state [12]. Consequently the energy dependence of the low-energy $S_{17}(E)$ factor is well-known (e.g. Refs. [13,11,14]). Its absolute value, however, depends on $\bar{c}$ and has thus to be determined experimentally. Nevertheless theoretical constraints on the asymptotic normalization constant might be quite useful. We note that $\bar{c}$ depends mainly on the effective ^{7}Be–p interaction radius. A larger radius results in a lower Coulomb barrier, which leads to a higher tunneling probability into the external region, and hence to a higher cross section. A possible way to constrain the interaction radius is to study some key properties of the $A=7$ and 8 nuclei [15]. The observables that are most sensitive to the interaction radius are "size-like" properties, for example, quadrupole moment, radius, Coulomb displacement energy [16], etc. These are the quantities which we will calculate in our microscopic cluster model and then study their effect on the astrophysical S factor.

II. THE SIZE OF ^{8}B AND ITS EFFECT ON S_{17}

In Ref. [9] we demonstrated that there is a linear correlation between the zero-energy astrophysical S factor, $S_{17}(0)$, of the ^{7}Be$(p,\gamma)^8$B reaction and the quadrupole moment of ^{7}Be,

Q_7. The Q_7 quadrupole moment has not been measured yet, but in Ref. [9] we have predicted it to be between -6 e fm^2 and -7 e fm^2. The absolute scale of the $S_{17}(0) - Q_7$ correlation, however, depends on the applied effective nucleon-nucleon ($N - N$) interaction. For our preferred MN interaction this resulted in $S_{17}(0) = 25 - 26.5$ eV b [9]. Other interactions gave slightly larger or smaller $S_{17}(0)$ values, but these interactions were found to be inferior to the MN force for other observables. Now we will turn to the other "size-like" observables and their potential constraints on $S_{17}(0)$. For this purpose we have repeated the microscopic calculations described in Ref. [9] varying the size parameter of the ^{3}He and ^{4}He clusters while keeping other important ingredients of the calculation fixed. Again as in Ref. [9] we have performed these calculations for several interactions (MN force [17], V2 interaction [18], and MHN interaction [19]). Our calculation thus reproduces the $S_{17}(0) - Q_7$ plot shown in Fig. 1 of Ref. [9].

In the present Fig. 1 we extend the study of Ref. [9] and investigate the relation of $S_{17}(0)$ to several size-like properties of ^{8}B: (a) the ^{8}B radius r(^{8}B), (b) the difference between the ^{7}Be and ^{8}B radii quantified by $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be})$, (c) the ^{8}B quadrupole moment, and (d) the $E(^8\mathrm{Li}) - E(^8\mathrm{B})$ Coulomb displacement energy. These calculations have been performed for the same model spaces and interactions as in Ref. [9]. Importantly all four indicators scale linearly with $S_{17}(0)$. This is caused by the 'halo' structure of the ^{8}B ground state [20] and reflects that the ^{7}Be(p,γ)^{8}B reaction at low energies is an external capture process. In the following we will discuss the four indicators in turn and will try to derive at possible constraints on $S_{17}(0)$. First, the comparison is performed for the consistent eight-body ^{4}He+^{3}He+p calculation using the MN force; the dependence of our results on the model space and the interaction employed will be discussed below.

The size property that is most sensitive to the effective $^7\mathrm{Be} - p$ interaction radius is $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be})$. However, a precise experimental determination of this quantity is very difficult. In fact, during the course of the work reported in Ref. [9] it seemed hopeless that the ^{7}Be or ^{8}B radius could be measured with relatively high precision. The radii of nuclei far from stability are usually extracted from interaction cross section measurements by using Glauber-type models with uniform density distribution for the nuclei [21]. Recently, a new and more precise method has been introduced [22] which considers the few-body structure of the nuclei involved (like $^7\mathrm{Be} + p$ for ^{8}B), while extracting the radius from the measured interaction cross sections. For ^{8}B the resulting point-nucleon radius is $r(^8\mathrm{B}) = 2.50 \pm 0.04$ fm, and hence $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be}) \approx 0.9$ fm^2. We note, that for ^{7}Be the model of Ref. [22] still uses the Glauber estimate. Our calculation reported in Ref. [9] gives $r(^8\mathrm{B}) = 2.73$ fm and thus overestimates the experimental value. As expected we observe that $S_{17}(0)$ decreases with decreasing ^{8}B radius. Using the linear relationship between $r(^8\mathrm{B})$ and $S_{17}(0)$ and the experimental value for the ^{8}B radius places the cross section into the range $S_{17}(0) = 23.2 - 24.2$ eV b. Ref. [9] found $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be}) = 0.8$ fm^2, which appears to be a reasonable value. However, due to the uncertainties in the phenomenological value the derivation of a constraint on $S_{17}(0)$ from $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be})$ is currently not possible.

The experimental value of the ^{8}B quadrupole moment is $Q_8 = (6.83 \pm 0.21)$ e fm^2 [23]. In Ref. [9] we calculated a slightly larger value, $Q_8 = 7.45$ fm^2. Like in the case of the ^{8}B radius, $S_{17}(0)$ increases linearly with the ^{8}B quadrupole moment. From a comparison to the experimental data we find the constraint $S_{17}(0) = 23.7 - 24.8$ eV b (Fig. 1c).

To derive a phenomenological value for the Coulomb displacement energy to be compared

with our calculated values, we have to consider that the physics that accounts for the Nolen-Schiffer effect [25] is not present in our model. This effect is estimated to cause a ≈130 keV shift in the $E(^8\text{Li}) - E(^8\text{B})$ Coulomb displacement [16]. So we should compare our results to a phenomenological value of $\Delta = 3.54 - 0.13 = 3.41$ MeV. Ref. [9] found a too small Coulomb displacement energy, $\Delta = 3.2$ MeV. From Fig. 1d we observe that $S_{17}(0)$ decreases linearly with Δ; thus the experimental value for the Coulomb displacement energy corresponds to $S_{17}(0) = 24.3$ eV b.

We can summarize the results for the complete ^{4}He+^{3}He+p cluster study (with the MN force) of Ref. [9] as follows: This calculation [9] gives consistently values for those indicators, for which reliable experimental data exist, which point to the use of a too large ^{7}Be+p interaction radius. This implies that the value for $S_{17}(0)$ (26.1 eV b) predicted in [9] is too large. We note that, by slightly varying the interaction radius, all three experimentally determined "size-like" parameters (the ^{8}B radius and quadrupole moment and the Coulomb displacement energy) can consistently be reproduced (see Table 1); the corresponding value for the ^{7}Be(p,γ)^{8}B reaction cross section then is $S_{17}(0) = (23 - 25)$ eV b.

How much do our results depend on the chosen model space and the adopted interaction? To answer these questions we have at first performed a series of restricted calculations involving only $(^3\text{He} + {}^4\text{He}) + p$ configurations ($^7\text{Be} + p$ like configurations) rather than all possible arrangements of the three clusters. (As in all calculations reported in this paper the experimental value of the ^{8}B binding energy relative to the $^7\text{Be} + p$ threshold has been reproduced by a slight modification in the $N - N$ interaction, see Ref. [9].) Again we find that $S_{17}(0)$ scales linearly with all indicators. From Fig. 1 we also observe that, for a fixed value of $S_{17}(0)$, the extension of the model space (going from the restricted space to the full three-cluster model) reduces the ^{8}B radius slightly, but increases the ^{8}B quadrupole moment and the Coulomb displacement energy. The reason why the radius (a) and the quadrupole moment (c) of ^{8}B change in opposite direction if the model space is enlarged is that the addition of the $^5\text{Li} + {}^3\text{He}$ and $^4\text{He} + {}^4\text{Li}$ channels brings in large charge polarization which increases the quadrupole moment even if $r(^8\text{B})$ is reduced. Even if we allow the variation of the ^{7}Be+p interaction radius, the restricted model space calculation does not simultaneously reproduce the experimental data for our indicators (see Table 1). While the ^{8}B radius and the Coulomb displacement energy puts $S_{17}(0)$ at around $23 - 24$ eV b, the ^{8}B quadrupole moment favors a larger value of $S_{17}(0) = 25.3 - 27.5$ eV b. In fact, the ^{8}B quadrupole moment is the quantity which is clearly most sensitive to the model space. Note that enlargening the model space does not only reduce $S_{17}(0)$ for fixed value of Q_8, it also changes the slope of the $S_{17}(0) - Q_8$ scaling. Obviously the reproduction of the ^{8}B quadrupole moment requires a 3-body approach and is sensitive to the internal structure of the clusters.

In Ref. [9] it has been observed that the $S_{17}(0) - Q_7$ scaling depends on the adopted NN interaction. We have therefore repeated the $^7\text{Be} + p$-type model calculations with two other interactions (V2 and MHN). We observe that, for fixed values of the ^{8}B radius, of the $r^2(^8\text{B}) - r^2(^7\text{Be})$ difference and of the Coulomb displacement, the rather repulsive V2 interaction gives larger $S_{17}(0)$ values than the MN interaction, while the MHN interaction gives smaller values. Assuming a linear relation betwen $S_{17}(0)$ and our indicators, constraints on the zero-energy S-factor can be derived; the corresponding values are listed in Table 1. For the MHN interaction we find that the ^{8}B quadrupole moment and the other indicators are not simultaneously reproduced in the restricted model space. For the V2 interaction the

^{8}B quadrupole moment and radius and the Coulomb displacement energy are reproduced for a ^{7}Be+p interaction radius corresponding to $S_{17}(0) \approx 26$ eV b. However, for this value of $S_{17}(0)$ the $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be})$ difference becomes unreasonably large.

We note again that a measurement of the ^{7}Be quadrupole moment would place some additional constraints on the consistency of our calculations. For the complete ^{4}He+^{3}He+p model calculation the simultaneous reproduction of the indicators predict Q_7 to be in the range $-(5.5-6.0)$ e fm^2. However, this value is smaller than the one ($Q_7 = -6.9$ e fm^2 [9]) obtained if we chose the cluster size parameters such to reproduce the quadrupole moment of the analog nucleus ^{7}Li. Does this already point to the necessity of a further enlargement of the model space beyond the $^4\mathrm{He} + {^3\mathrm{He}} + p$ three-cluster model which would then also effect our results obtained for ^{7}Be, e.g., change the ^{7}Be quadrupole moment? To investigate this we have performed calculations for ^{7}Be in which we have added the $^6\mathrm{Li}+p = {^4\mathrm{He}}+d+p$ configuration to the ^{4}He+^{3}He model space, adopted above. In all cases the exchange mixture parameter of the $N-N$ interaction was fixed to reproduce the ^{7}Be binding energy relative to the $^4\mathrm{He} + {^3\mathrm{He}}$ threshold. We also made sure that the $^6\mathrm{Li} + p$ threshold was correctly reproduced. Our results show that $|Q_7|$ is increased by $0.5-1$ e fm^2 in the coupled-channel model relative to the single-channel $^4\mathrm{He} + {^3\mathrm{He}}$ value perhaps suggesting the need for an even larger model space than the complete ^{4}He+^{3}He+p model space adopted here. Which consequences such an enlargement might have on $S_{17}(0)$ has to wait for ^{8}B calculations performed in larger model spaces, which are beyond $^4\mathrm{He} + {^3\mathrm{He}} + p$.

III. SUMMARY

In summary, we have adopted a microscopic $^4\mathrm{He} + {^3\mathrm{He}} + p$ cluster model to calculate the ^{8}B radius and quadrupole moment, the difference in the ^{7}Be and ^{8}B radii, $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be})$, and the Coulomb displacement energy $E(^8\mathrm{Li}) - E(^8\mathrm{B})$ and to study their relation to the $S_{17}(0)$ astrophysical S factor. We find that all these indicators scale linearly with the zero-energy ^{7}Be(p,γ)^{8}B cross section.

Within our three-cluster model we find that the experimentally determined values for the ^{8}B radius and quadrupole moment as well as the Coulomb displacement energy is consistently described if the internal cluster size parameters are chosen such that the values for $S_{17}(0)$ are between $(23-25)$ eV b. This range is more or less compatible with the value currently used in most solar models, $S_{17}(0) = 22.4 \pm 2.1$ eV b [26]. However, it is slightly inconsistent with the recently adopted new experimental value $S_{17}(0) = 19^{+4}_{-2}$ eV b [27].

As a note of caution we mention that this result has been derived from a linear relation between our four indicators and $S_{17}(0)$ found in our three-cluster model. However, we found that enlarging the ^{7}Be model space by adding a $^4\mathrm{He} + d + p$ configuration to the $^4\mathrm{He} + {^3\mathrm{He}}$ configuration increased the ^{7}Be quadrupole moment by about 10%. To investigate the effects which additional configuration might have on the ^{8}B properties and in particular on the $S_{17}(0)$ value, requires calculations in model spaces which are beyond $^4\mathrm{He} + {^3\mathrm{He}} + p$.

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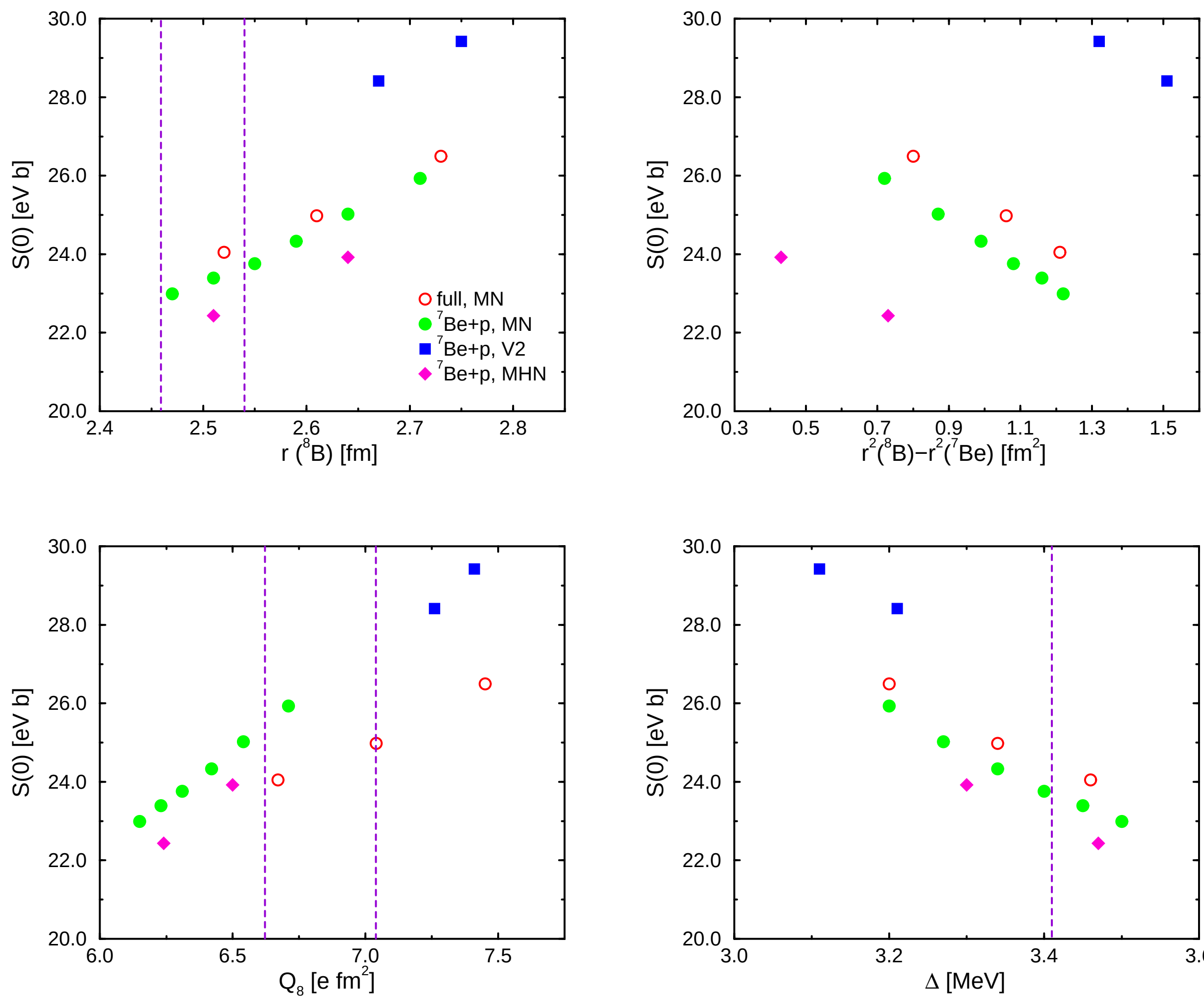
TABLES

TABLE I. The constraints derived for $S_{17}(0)$ (in eV b) from the ^{8}B radius and quadrupole moment and from the Coulomb displacement energy in our complete ^{4}He+^{3}He+p 8-body calculation (full) and in the restricted ^{7}Be+p model spaces for the Minnesota force (MN), the Volkov force (V2) and the Hasegawa-Nagata force (MHN).

indicator	full	MN	V2	MHN
r(^{8}B)	$23.2-24.2$	$22.8-23.6$	$25.7-26.6$	$21.7-22.7$
Q_8	$23.7-24.8$	$25.3-27.5$	$24.1-27.0$	$24.3-27.2$
Δ	24.3	23.8	26.5	23.0

FIGURES

FIG. 1. Correlation between the zero-energy astrophysical S factor of the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reaction and (a) the ^{8}B point-nucleon radius, $r(^8\mathrm{B})$ (in fm), (b) the $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be})$ value (in fm^2), (c) the quadrupole moment of ^{8}B, Q_8 (in e fm^2), and (d) the $\Delta = E(^8\mathrm{Li}) - E(^8\mathrm{B})$ Coulomb displacement energy (in MeV). The correlations have been calculated in our microscopic eight-body model, using several $N - N$ interactions and model spaces. For a given model space and interaction, different results are obtained by varying the cluster size parameters. For a detailed description of the model spaces and interactions, see Ref. [9]. The phenomenological values are $r(^8\mathrm{B}) = 2.50 \pm 0.04$ fm [22], $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be}) \approx 0.9$ fm^2, $Q_8 = 6.83 \pm 0.21$ e fm^2 [23], and $\Delta = 3.41$ MeV. In the Coulomb displacement energy the Nolen-Schiffer anomaly is removed from the value given in Ref. [24] (see the text). The phenomenological values are indicated by dashed lines in the respective figures.



S(0) [eV b]
r (^{8}B) [fm]
full, MN
^{7}Be+p, MN
^{7}Be+p, V2
^{7}Be+p, MHN
$r^2(^8\text{B})-r^2(^7\text{Be})$ [fm^2]
Q_8 [e fm^2]
Δ [MeV]